## Electric Potential

P21.15. Strategize: We will use what we know about the potential due to point charges. The potential due to the point charges is just the sum of the potentials due to each charge individually.
Prepare: Let the distance between the two charges be called $2 r$. Then the distance from the observation point midway between the charges to either charge is $r$. The potential near a charged particle is given by Equation 21.11: $V=K Q / r$.


Solve: The total potential midway between the charges is equal to the sum of the potential of either charge by itself. Thus we have:

$$
300 \mathrm{~V}=\frac{K Q}{r}+\frac{K Q}{r}=\frac{2 K Q}{r} \Rightarrow \frac{K Q}{r}=150 \mathrm{~V}
$$

We don't know $Q$ or $r$ but this ratio of $K Q$ and $r$ is all we need to solve the problem. Since the total distance between the charges is $2 r$, a point $25 \%$ of the way from one particle to the other is a distance $r / 2$ from the closer particle and a distance $3 r / 2$ from the farther particle. Thus the potential at such a point is:

$$
V=\frac{K Q}{r / 2}+\frac{K Q}{3 r / 2}=\frac{8 K Q}{3 r}=\left(\frac{8}{3}\right)(150 \mathrm{~V})=400 \mathrm{~V}
$$

Assess: Going from the midpoint of the two charges to a point closer to one of them increases the potential. This means that we would have to do work to move a positive test charge from the midpoint toward one of the charges. This makes sense considering that very close to either charge, the field is strong and a positive test charge placed there would experience a strong repulsive force.

P21.16. Strategize: We will use the relationship between the electric field, distance, and potential difference for the first part. For the second, we will use the definition of capacitance, which is the charge per that can be held by a device per unit of potential difference applied to the device. We will also calculate the capacitance using its geometry.
Prepare: The electric potential difference between the plates is determined by the uniform electric field in the parallel-plate capacitor and is given by Equation 21.6.
Solve: (a) The potential difference $\Delta V_{\mathrm{C}}$ across a capacitor of spacing $d$ is related to the electric field inside by Equation 21.6:

$$
E=\frac{\Delta V_{\mathrm{C}}}{d} \Rightarrow \Delta V_{\mathrm{C}}=E d=\left(1.0 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)(0.002 \mathrm{~m})=200 \mathrm{~V}
$$

(b) The electric field of a capacitor is related to the charge by Equation 20.7:

$$
Q=\grave{\mathrm{o}}_{0} A E=\left(8.85 \times 10^{-12} \mathrm{C}^{2} /\left(\mathrm{N} \cdot \mathrm{~m}^{2}\right)\right)\left(4.0 \times 10^{-4} \mathrm{~m}^{2}\right)\left(1.0 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)=3.5 \times 10^{-10} \mathrm{C}
$$

Assess: A charge of 0.35 nC on the positive plate and an equal negative charge on the negative plate create a significant potential difference across the parallel.

P21.17. Strategize: We know how to relate the potential difference to the electric field in a capacitor. We also know an expression for the electric field between two charged plates in terms of the charge density. We can combine to solve for the potential difference.
Prepare: The electric potential between the plates of a parallel plate capacitor is determined by the uniform electric field between the plates by Equation 21.6.
Solve: (a) Using Equations 21.6 and 20.7, the potential difference across the plates of a capacitor is

$$
\Delta V_{\mathrm{C}}=E d=\frac{(Q / A)}{\grave{\mathrm{o}}_{0}} d=\frac{Q d}{A \grave{o}_{\mathrm{o}}}=\frac{\left(0.708 \times 10^{-9} \mathrm{C}\right)\left(1.0 \times 10^{-3} \mathrm{~m}\right)}{\left(4.0 \times 10^{-4} \mathrm{~m}^{2}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} /\left(\mathrm{N} \cdot \mathrm{~m}^{2}\right)\right.}=200 \mathrm{~V}
$$

Then the electric field is $\Delta V_{\mathrm{C}} / d=(200 \mathrm{~V}) /\left(1.0 \times 10^{-3} \mathrm{~m}\right)=2.0 \times 10^{5} \mathrm{~N} / \mathrm{C}$.
(b) For $d=2.0 \mathrm{~mm}, \Delta V_{\mathrm{C}}=400 \mathrm{~V}$ and the electric field is $\Delta V_{\mathrm{C}} / d=(400 \mathrm{~V}) /\left(2.0 \times 10^{-3} \mathrm{~m}\right)=2.0 \times 10^{5} \mathrm{~N} / \mathrm{C}$.

Assess: These answers make sense: We know that the electric field inside a parallel plate capacitor with a fixed charge does not depend on the plate separation.

P21.18. Strategize: Consider which types of charge cause a positive or large potential, and which types of charge cause a negative potential, or reduce the potential. Given potential difference and distance we can determine the electric field. For the last part, we relate potential and electric potential energy using the charge of the proton.
Prepare: The electric field inside a parallel-place capacitor is determined by the potential difference between the plates given by Equation 21.7. The proton's potential energy inside the capacitor is also determined by the capacitor's potential difference.
Solve: (a) Because the right plate is at a higher potential compared with the left plate, the positive plate is on the right and has a potential of 300 V .
(b) The electric field strength inside the capacitor is

$$
E=\frac{\Delta V_{\mathrm{C}}}{d}=\frac{300 \mathrm{~V}-0 \mathrm{~V}}{3.0 \times 10^{-3} \mathrm{~m}}=1.0 \times 10^{5} \mathrm{~V} / \mathrm{m}
$$

(c) The potential energy of a charge $q$ is $U=q \Delta V$. A proton on the left plate will have zero potential energy. A proton at the midpoint of the capacitor is at a potential of 150 V . Thus, its potential energy is

$$
U=\left(1.6 \times 10^{-19} \mathrm{C}\right)(150 \mathrm{~V})=2.4 \times 10^{-17} \mathrm{~J}
$$

Assess: Because the right plate is at a higher potential compared with the left plate, the proton's potential energy at midpoint was expected to be positive.

P21.19. Strategize: This problem involves the concept of equipotential lines.
Prepare: The charge is a point charge. We will use Equations 21.10 and 21.1 to calculate the potential at each point.
Solve: The potentials are given by $V_{\mathrm{A}}=V_{\mathrm{B}}=\frac{K q}{r}=\frac{\left(2.0 \times 10^{-9} \mathrm{C}\right)\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}{0.01 \mathrm{~m}}=1800 \mathrm{~V}$.
For point C, $r=0.02 \mathrm{~m}$ and $V_{\mathrm{C}}=900 \mathrm{~V}$. The potential differences are

$$
\Delta V_{\mathrm{AB}}=V_{\mathrm{B}}-V_{\mathrm{A}}=1800 \mathrm{~V}-1800 \mathrm{~V}=0 \mathrm{~V} \quad \Delta V_{\mathrm{BC}}=V_{\mathrm{C}}-V_{\mathrm{B}}=900 \mathrm{~V}-1800 \mathrm{~V}=-900 \mathrm{~V}
$$

Assess: Clearly $V_{\mathrm{A}}=V_{\mathrm{B}}$ and $V_{\mathrm{C}}$ and $V_{\mathrm{A}}$, so, as expected, $\Delta V_{\mathrm{AB}}=0$ and $\Delta V_{\mathrm{BC}}$ is negative.

P21.21. Strategize: The potential due to many point charges is just the sum of the potential due to each.
Prepare: The potential is given by Equation 21.10.
Solve: The potential at the dot is

$$
V=\frac{1}{4 \pi \grave{\mathrm{o}}_{0}} \frac{q_{1}}{r_{1}}+\frac{1}{4 \pi \grave{\mathrm{o}}_{0}} \frac{q_{2}}{r_{2}}+\frac{1}{4 \pi \grave{\mathrm{o}}_{0}} \frac{q_{3}}{r_{3}}=\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left[\frac{2.0 \times 10^{-9} \mathrm{C}}{0.040 \mathrm{~m}}+\frac{2.0 \times 10^{-9} \mathrm{C}}{0.050 \mathrm{~m}}+\frac{2.0 \times 10^{-9} \mathrm{C}}{0.030 \mathrm{~m}}\right]=+1400 \mathrm{~V}
$$

Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

P21.27. Strategize: We know the electric field points from regions of higher potential to regions of lower potential. We also know the relationship between the electric field strength, the potential difference, and distance.
Prepare: In a region that has a uniform electric field, Equation 21.17 gives the magnitude of the potential difference between two points.
Solve: (a) The electric field points "downhill." So, point A is at a higher potential than point B.
(b) The magnitude of the potential difference between points A and B is

$$
\Delta V=E_{x} d=(1000 \mathrm{~V} / \mathrm{m})(0.07 \mathrm{~m})=70 \mathrm{~V}
$$

That is, the potential at point A is 70 V higher than the potential at point B .
Assess: Electric field points from higher potential to lower potential.
P21.29. Strategize: We can use Equation 21.17 to determine the magnitude of the potential difference. The sign is given by considering that electric fields point from higher potentials to lower ones.
Prepare: In order to use Equation 21.17, $E=\Delta V / d$, we need to know the shortest distance between two equipotential surfaces. We are given the distance between point A which is on one surface and point B which is on another surface. However this is not the shortest distance, as you can see from the figure. The shortest distance, $d$, is given by $d=(5.0 \mathrm{~cm}) \cos 30^{\circ}=2.5 \sqrt{3} \mathrm{~cm}$.


Solve: Since the electric field vectors point from the surface containing A to the surface containing $\mathrm{B}, \mathrm{A}$ is at a higher potential and $V_{\mathrm{A}}-V_{\mathrm{B}}$ will be a positive potential difference. From Equation 21.17, the potential difference is given by:

$$
V_{\mathrm{A}}-V_{\mathrm{B}}=E d=(1200 \mathrm{~V} / \mathrm{m})\left(2.5 \sqrt{3} \times 10^{-2} \mathrm{~m}\right)=52 \mathrm{~V}
$$

Given that $V_{\mathrm{A}}=-300 \mathrm{~V}$, we conclude that $V_{\mathrm{B}}=-352 \mathrm{~V}$.

Assess: Even though the electric field vectors do not point directly from point A to point B, the potential is less at B than at A because the displacement vector from A to B forms an acute angle, $30^{\circ}$, with the electric field. That means that to get from A to B , one component of your motion must be in the direction of the field.

P21.60. Strategize: We calculate the potential energy at the two points using Equation 21.10 , and take the difference.
Prepare: The fixed 25.0 nC point charge creates a potential at point A and a different value at point B . We'll first use Equation 21.10 to calculate the potential difference due to the 25.0 nC point charge between points A and B .

$$
V=K \frac{q}{r}
$$

where $q=25.0 \mathrm{nC}$ and $r_{\mathrm{A}}=0.050 \mathrm{~m}$ and $r_{\mathrm{B}}=0.015 \mathrm{~m}$.
Then we'll use $\Delta U_{\text {elec }}=q \Delta V$.
Solve:

$$
\begin{aligned}
\Delta V_{\mathrm{AB}} & =V_{\mathrm{B}}-V_{\mathrm{A}}=K \frac{q}{r_{\mathrm{B}}}-K \frac{q}{r_{\mathrm{A}}}=K q\left(\frac{1}{r_{\mathrm{B}}}-\frac{1}{r_{\mathrm{B}}}\right) \\
& =\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)(25.0 \mathrm{nC})\left(\frac{1}{0.015 \mathrm{~m}}-\frac{1}{0.050 \mathrm{~m}}\right)=10500 \mathrm{~V}
\end{aligned}
$$

Now we move the 3.0 nC charge from point A to B .

$$
\left(\Delta U_{\text {elec }}\right)_{\mathrm{AB}}=q \Delta V_{\mathrm{AB}}=(3.0 \mathrm{nC})(10500 \mathrm{~V})=3.15 \times 10^{-5} \mathrm{~J} \approx 3.2 \times 10^{-5} \mathrm{~J}
$$

Assess: It doesn't really matter that the line from the 25.0 nC charge to point A is at a right angle with the line to point B; they could all be colinear (or any other angle) and we'd still get the same answer.

P21.64. Strategize: We can write an expression for the sum of the two charges, and for the product of the two charges (using Equation 21.9). We can solve the system of two equations for the two unknown charges.
Prepare: Let the unknown charges be $Q_{1}$ and $Q_{2}$, then $Q_{1}+Q_{2}=30 \times 10^{-9} \mathrm{C}$. Equation 21.9 for the electric potential energy reads $U=K \frac{Q_{1} Q_{2}}{r_{12}}$ or $U=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{1} Q_{2}}{r_{12}}$.
Solve:

$$
U=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{1} Q_{2}}{r_{12}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{1} Q_{2}}{2.0 \times 10^{-2} \mathrm{~m}}=-180 \times 10^{-6} \mathrm{~J} \Rightarrow Q_{1} Q_{2}=\frac{-\left(180 \times 10^{-6} \mathrm{~J}\right)\left(2.0 \times 10^{-2} \mathrm{~m}\right)}{9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}=-4.0 \times 10^{-16} \mathrm{C}^{2}
$$

Solving the first equation for $Q_{2}$ and substituting into the second equation,

$$
\begin{aligned}
Q_{1}\left(30 \times 10^{-9} \mathrm{C}-Q_{1}\right) & =-4.0 \times 10^{-16} \mathrm{C}^{2} \Rightarrow Q_{1}^{2}-\left(30 \times 10^{-9} \mathrm{C}\right) Q_{1}-\left(4.0 \times 10^{-16} \mathrm{C}^{2}\right)=0 \\
& \Rightarrow Q_{1}=\frac{\left(30 \times 10^{-9} \mathrm{C}\right) \pm \sqrt{\left(-30 \times 10^{-9} \mathrm{C}\right)^{2}+4\left(4.0 \times 10^{-16} \mathrm{C}^{2}\right)}}{2} \Rightarrow Q_{1}=+40 \mathrm{nC} \text { and }-10 \mathrm{nC}
\end{aligned}
$$

That is, the two charges are -10 nC and 40 nC .
Assess: As they must, the two charges when added yield a total charge of 30 nC , and when substituted into the potential energy equation yield $U=-180 \times 10^{-6} \mathrm{~J}$.

P21.66. Strategize: Let us symbolically write the potential from each of the given charges at an unknown location. Then we require that the sum of the two potential contributions be zero, and solve for the location.
Prepare: The net potential is the sum of the scalar potentials due to each charge. Let the point on the $y$-axis where the electric potential is zero be at a distance $y$ from the origin. At this point, $V_{1}+V_{2}=0 \mathrm{~V}$.

P21.71. Strategize: The net potential at the dot is the sum of the potentials due to each charge.

Prepare: The dot is equidistant from the three charges. We will denote the 2 nC charge as 1 , left -1 nC charge as 2 , and the right -1 nC charge as 3 . From the geometry in the figure,

$$
\frac{1.5 \mathrm{~cm}}{r_{1}}=\frac{1.5 \mathrm{~cm}}{r_{2}}=\frac{1.5 \mathrm{~cm}}{r_{3}}=\cos 30^{\circ} \Rightarrow r_{1}=r_{2}=r_{3}=\frac{1.5 \mathrm{~cm}}{\cos 30^{\circ}}=1.732 \mathrm{~cm}
$$

Solve: The potential at the dot is

$$
V=\frac{1}{4 \pi \grave{\mathrm{o}}} \frac{q_{1}}{r_{1}}+\frac{1}{4 \pi \grave{\mathrm{o}}_{\mathrm{b}}} \frac{q_{2}}{r_{2}}+\frac{1}{4 \pi \grave{\mathrm{o}}_{\mathrm{i}}} \frac{q_{3}}{r_{3}}=\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left[\frac{2.0 \times 10^{-9} \mathrm{C}}{0.01732 \mathrm{~m}}-\frac{1.0 \times 10^{-9} \mathrm{C}}{0.01732 \mathrm{~m}}-\frac{1.0 \times 10^{-9} \mathrm{C}}{0.01732 \mathrm{~m}}\right]=0.0 \mathrm{~V}
$$

Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.


Solve: Using Equation 21.10,

$$
\begin{aligned}
\frac{1}{4 \pi \varepsilon_{0}}\left\{\frac{q_{1}}{r_{2}}+\frac{q_{2}}{r_{2}}\right\} & =0 \mathrm{~V} \Rightarrow \frac{-3.0 \times 10^{-9} \mathrm{C}}{\sqrt{(-9.0 \mathrm{~cm})^{2}+y^{2}}}+\frac{4.0 \times 10^{-9} \mathrm{C}}{\sqrt{(16.0 \mathrm{~cm})^{2}+y^{2}}}=0 \\
& \Rightarrow 3 \sqrt{(16 \mathrm{~cm})^{2}+y^{2}}=4 \sqrt{(-9 \mathrm{~cm})^{2}+y^{2}} \Rightarrow 9\left(256 \mathrm{~cm}^{2}+y^{2}\right)=16\left(81 \mathrm{~cm}^{2}+y^{2}\right) \\
& \Rightarrow 7 y^{2}=1008 \mathrm{~cm}^{2} \Rightarrow y= \pm 12 \mathrm{~cm} .
\end{aligned}
$$

Assess: In comparison to separation between the charges, these values seem reasonable.
P21.74. Strategize: Let us assume no forces other than electric forces act on the proton. Then the lost electric potential energy is accounted for by an increase in kinetic energy.
Prepare: Because the small positive charge moves farther away from the large negative charge the electric potential energy increases. The kinetic energy of the small charge decreases by the same amount.
Solve: Use conservation of energy.

$$
\begin{aligned}
K_{\mathrm{i}} & +U_{\mathrm{i}}=K_{\mathrm{f}}+U_{\mathrm{f}} \Rightarrow \\
K_{\mathrm{f}} & =K_{\mathrm{i}}+\left(U_{\mathrm{i}}-U_{\mathrm{f}}\right)=\frac{1}{2} m v_{\mathrm{i}}^{2}+\frac{1}{4 \pi \grave{\mathrm{o}}_{\mathrm{o}}} Q_{1} Q_{2}\left(\frac{1}{r_{\mathrm{i}}}-\frac{1}{r_{\mathrm{f}}}\right) \\
& =\frac{1}{2}\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(1.9 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)^{2}+\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)(-10 \mathrm{nC})\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(\frac{1}{3 \mathrm{~mm}}-\frac{1}{4 \mathrm{~mm}}\right) \\
& =1.81 \times 10^{-15} \mathrm{~J}
\end{aligned}
$$

Now solve for the final speed.

$$
v_{\mathrm{f}}=\sqrt{\frac{2 K_{\mathrm{f}}}{m}}=\sqrt{\frac{2\left(1.81 \times 10^{-15} \mathrm{~J}\right)}{1.67 \times 10^{-27} \mathrm{~kg}}}=1.5 \times 10^{6} \mathrm{~m} / \mathrm{s}
$$

Assess: We expected the new speed to be smaller than the old speed, but in the same ballpark, and it is.

P21.75. Strategize: Assuming the system of the proton and the plates is isolated, the total energy will be conserved. We can find the change in kinetic energy by finding the change in electric potential energy.
Prepare: The proton will be attracted to the upper, negative plate and repelled by the lower positive plate. Since it is fired in halfway between the plates, by the time it hits the upper plate it will have moved through a potential difference of $\Delta V=2500 \mathrm{~V}$, so it will have gained 2500 eV of kinetic energy. We convert eV to J : $2500 \mathrm{eV}=4.0 \times 10^{-16} \mathrm{~J}$.
Solve: Use an energy equation.

$$
K_{\mathrm{f}}=K_{\mathrm{i}}+\Delta K=\frac{1}{2} m v_{\mathrm{i}}^{2}+\Delta K=\frac{1}{2}\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(3.0 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)^{2}+4.0 \times 10^{-16} \mathrm{~J}=4.75 \times 10^{-16} \mathrm{~J}
$$

Now solve for the final speed.

$$
v_{\mathrm{f}}=\sqrt{\frac{2 K_{\mathrm{f}}}{m}}=\sqrt{\frac{2\left(4.75 \times 10^{-16} \mathrm{~J}\right)}{1.67 \times 10^{-27} \mathrm{~kg}}}=7.5 \times 10^{5} \mathrm{~m} / \mathrm{s}
$$

Assess: We expected the final speed to be greater than the initial speed, but in the same ballpark, and it is.

P21.78. Strategize: We can determine the voltage from the electric field and plate spacing using Equation 21.6. For the second part the capacitance can be determined from the given geometry, and the capacitance and voltage determine the charge on the plates. For the final part, we'll use conservation of energy.
Prepare: The electron has charge $q=-e$, and its potential energy at a point where the capacitor's potential is $V$ is $U=-e V$. Since the electron is launched from the negative (lower potential) plate toward the positive (higher potential) plate, its potential energy becomes more negative (because of the negative sign of the electron charge). That is, the potential energy decreases, which must lead to an increase in the kinetic energy. Conversely, the electron's speed as it is launched is smaller than $2.0 \times 10^{7} \mathrm{~m} / \mathrm{s}$. Energy is conserved. The electron's potential energy inside the capacitor can be found from the capacitor's electric potential.
Solve: (a) From Equation 21.6, the voltage across the capacitor is

$$
\Delta V_{\mathrm{C}}=E d=\left(5.0 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)\left(2.0 \times 10^{-3} \mathrm{~m}\right)=1000 \mathrm{~V}
$$

(b) Because $E=\Delta V_{C} / d, Q=C \Delta V_{C}$, and $C=\grave{o}_{0} A / d$, so $E=Q / \grave{o}_{0} A$. Thus, the charge on each plate is

$$
Q=\pi R^{2} E \grave{o}_{0}=\pi\left(1.0 \times 10^{-2} \mathrm{~m}\right)^{2}\left(5.0 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)=1.4 \times 10^{-9} \mathrm{C}
$$

(c) The conservation of energy equation is

$$
\begin{aligned}
K_{\mathrm{f}}+q V_{\mathrm{f}}=K_{\mathrm{i}}+q V_{\mathrm{i}} & \Rightarrow \frac{1}{2} m v_{\mathrm{i}}^{2}=\frac{1}{2} m v_{\mathrm{f}}^{2}+q\left(V_{\mathrm{f}}-V_{\mathrm{i}}\right) \Rightarrow v_{\mathrm{i}}^{2}=v_{\mathrm{f}}^{2}+\frac{2}{m}(-e)(1000 \mathrm{~V}) \\
& \Rightarrow v_{\mathrm{i}}=\sqrt{\left(2.0 \times 10^{7} \mathrm{~m} / \mathrm{s}\right)^{2}-\frac{2\left(1.60 \times 10^{-19} \mathrm{C}\right)(1000 \mathrm{~V})}{9.11 \times 10^{-31} \mathrm{~kg}}}=7.0 \times 10^{6} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

P21.80. Strategize: We can use energy conservation for the first part. Once we have the initial speed we can use that to determine the kinetic energy at half that speed, and use energy conservation again for part (b). For the acceleration, we will use Newton's $2^{\text {nd }}$ law.
Prepare: As the electron moves toward the sphere, the only force acting on it is the Coulomb force, which is a conservative force. As a result, energy is conserved. At the point where the electron is stopped and reflected back, all its initial kinetic energy $\left(K E=m v^{2} / 2\right)$ has been converted into electric potential energy ( $U=k e Q / r$ ). Once we know where the electron is momentarily stopped, we can find the electric field at this point, the force on the electron at this point, and finally, the electron's acceleration at this point.
Solve: (a) Knowing that energy is conserved, we can write that the total initial energy is equal to the total final energy: $K_{\mathrm{i}}+U_{\mathrm{i}}=K_{\mathrm{f}}+U_{\mathrm{f}}$.
Since the electron is fired from far away, its initial electric potential energy is zero. Since the electron moves toward the sphere until it is momentarily stopped, the final kinetic energy is zero. This is written as $K E_{\mathrm{i}}=U_{\mathrm{f}}$.

Inserting expressions for the kinetic end electric potential energy obtain $m v^{2} / 2=k e Q / r$. This may by solved for $v$ to obtain $v=\sqrt{2 k e Q /(r m)}$.
Note that r in this expression is the radius of the sphere $(1.25 \mathrm{~mm})$ plus the distance the electron stops from the surface of the sphere $(0.30 \mathrm{~mm})$ or $r=1.55 \mathrm{~mm}$. Inserting values obtain $v$.

$$
v=\sqrt{2\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(4.5 \times 10^{-9} \mathrm{C}\right) /\left(\left(1.55 \times 10^{-3} \mathrm{~m}\right)\left(9 \times 10^{-31} \mathrm{~kg}\right)\right)}=9.6 \times 10^{7} \mathrm{~m} / \mathrm{s}
$$

(b) When the speed of the electron is half its original value, its kinetic energy is one fourth its original value. This is because kinetic energy is proportional to the square of speed. Let us call the distance from the center of the charge to the electron at this half-original-speed point, $r^{\prime}$. Since the total energy of the electron is equal to its initial kinetic energy, that is, $E_{\text {total }}=K_{\mathrm{i}}$ and since its kinetic energy at the point in question is $1 / 4$ times its original value, we can use conservation of energy to say:

$$
E_{\text {total }}=K_{\mathrm{i}}=\frac{1}{4} K_{\mathrm{i}}+U\left(r^{\prime}\right) \Rightarrow U\left(r^{\prime}\right)=\frac{3}{4} K_{\mathrm{i}}=\frac{3}{4} U_{\mathrm{f}}
$$

Now using the formula for the potential energy we can solve for $r^{\prime}$ :

$$
U\left(r^{\prime}\right)=\frac{k e Q}{r^{\prime}}=\frac{3}{4} U_{\mathrm{f}}=\frac{3}{4} \frac{\mathrm{keQ}}{r} \Rightarrow r^{\prime}=\frac{4 r}{3}=\frac{4(1.55 \mathrm{~mm})}{3}=2.07 \mathrm{~mm}
$$

When the electron has this distance from the center of the charge, its distance from the surface of the charge is $2.07 \mathrm{~mm}-1.25 \mathrm{~mm}=0.82 \mathrm{~mm}$.
(c) The magnitude of the electric field at the turning point may be obtained by $E=k Q / r^{2}$. The force on the electron at the turning point is obtained by $F=e E$. Finally, the acceleration of the electron at its turnaround point is

$$
a=F / m=e E / m=e\left(k Q / r^{2}\right) / m=k e Q /\left(m r^{2}\right)
$$

Inserting values

$$
a=\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(4.5 \times 10^{-9} \mathrm{C}\right) /\left(\left(9.0 \times 10^{-31} \mathrm{~kg}\right)\left(1.55 \times 10^{-3} \mathrm{~m}\right)^{2}\right)=3.0 \times 10^{18} \mathrm{~m} / \mathrm{s}^{2}
$$

Assess: The electron is fired at the sphere with a speed that is about one third the speed of light. If the speed were much greater we would have to consider relativistic effects. The position where the electron has half the speed should be larger than the turnaround point, and it is four times larger. Finally, the acceleration is very large, but this is acceptable since the mass of the electron is so small.

P21.81. Strategize: We can use energy conservation to equate the magnitude of the change in electric potential energy to that of the kinetic energy. Then we can solve for the unknown initial speed.
Prepare: The proton is fired from a distance much greater than the nuclear diameter, so $r_{1} \approx \infty$ and $U_{\mathrm{i}} \approx 0 \mathrm{~J}$. Because the nucleus is so small, a proton that is even a few atoms away is, for all practical purposes, at infinity. As the proton approaches the nucleus, it is slowed by the repulsive electric force. At the end point, the proton has just reached the surface of the nucleus ( $r_{\mathrm{f}}=$ nuclear diameter) with $v_{\mathrm{f}}=0 \mathrm{~m} / \mathrm{s}$. (The proton won't remain at this point but will be pushed back out again, but the subsequent motion is not part of this problem.) Initially, the proton has kinetic energy but no potential energy. At the point of closest approach, where $v_{\mathrm{f}}=0 \mathrm{~m} / \mathrm{s}$, the proton has potential energy but no kinetic energy. Energy is conserved. Because the iron nucleus is very large compared to the proton, we will assume that the nucleus does not move (no recoil) and that the proton is essentially a point particle with no diameter.


Solve: Because energy is conserved, $K_{\mathrm{f}}+U_{\mathrm{f}}=K_{\mathrm{i}}+U_{\mathrm{i}}$. This equation is

$$
0 \mathrm{~J}+\frac{(e)(26 e)}{4 \pi \grave{\mathrm{o}}_{0} r_{\mathrm{f}}}=\frac{1}{2} m_{\text {proton }} v_{\mathrm{i}}^{2}+0 \mathrm{~J}
$$

where $r_{\mathrm{f}}$ is half the nuclear diameter. The initial speed of the proton is

$$
v_{\mathrm{i}}=\sqrt{\frac{2(e)(26 e)}{4 \pi \mathrm{o}_{0} r_{\mathrm{f}} m_{\text {proton }}}}=\sqrt{\frac{2\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(26 \times 1.6 \times 10^{-19} \mathrm{C}\right)\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}{\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(4.5 \times 10^{-15} \mathrm{~m}\right)}}=4.0 \times 10^{7} \mathrm{~m} / \mathrm{s}
$$

Assess: Extremely large deceleration of the proton occurs as the proton is brought to rest momentarily.
P21.82. Strategize: In both cases, the electric field strength is related to the rate at which the potential is changing over distance.
Prepare: We will use Equation 21.17.
Solve: The contours are uniformly spaced along the $y$-axis above and below the origin. Point 1 is in the center of a 50 V change ( 25 V to 75 V ) over a distance of 2 cm , so the magnitude of the electric field using Equation 21.17 is $E=\Delta V / d$ is $(50 \mathrm{~V}) /(2 \mathrm{~cm})=25 \mathrm{~V} / \mathrm{cm}$ or $2500 \mathrm{~V} / \mathrm{m}$. Point 2 has the same potential difference in half the distance.
Thus the magnitude of the electric field at point 2 is $5000 \mathrm{~V} / \mathrm{m}$.
The magnitudes of the electric fields at points 1 and 2 are $2500 \mathrm{~V} / \mathrm{m}$ and $5000 \mathrm{~V} / \mathrm{m}$. The directions of the electric fields are downward at point 1 and upward at point 2 , that is, from the higher potential to the lower potential.
That is,

$$
\vec{E}_{1}=(2500 \mathrm{~V} / \mathrm{m}, \text { down }) \quad \vec{E}_{2}=(5000 \mathrm{~V} / \mathrm{m}, \text { up })
$$

