DC Resistive Circuits (an example report)

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September 8, 2021

Abstract: Several resistive circuits were both modeled in LTSpice and physically constructed. The measurements made on the physical circuits demonstrated behaviors that were consistent with Kirchoff's voltage and current laws, as well as demonstrating how resistors in series and parallel combine. A more complex network of power supplies and resistors was modeled, built and measured to demonstrate Thevenin's theorem. Finally, a light emitting diode and a photoresistor were used to demonstrate an optically coupled isolation circuit.

1. Series Resistors and KVL

The circuits shown in Figure 1A and 1B were modeled in LTSpice[1] and a DC operating point simulation was run.

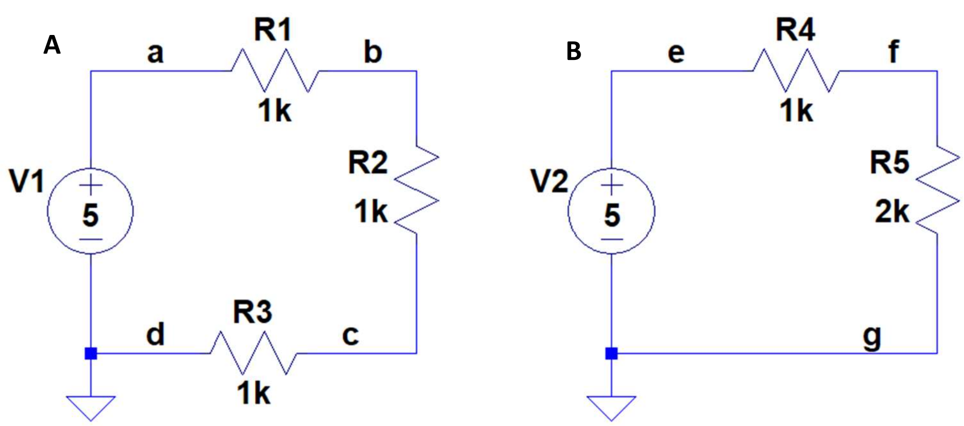

Figure 1 (A) Three series resistor circuit, (B) two series resistor circuit.

In the simulation the same current of $1.\overline{666}$ mA ran through each circuit and the potentials at node b in circuit A was the same as that at node f in circuit B ($3.\overline{333}$ V). These results show that the series pair of 1kΩ resistors in circuit A is equivalent to the single 2kΩ resistor in circuit B, consistent with the idea that series resistances combine to an equivalent single resistance given by the sum of the individual resistances.[2]

In considering circuit A in greater detail, the voltage at node a was 5.000V (the source voltage), at node b, it was 3.333 V, at node c it was 1.667 V and at node d it was at ground (0V). This is consistent with KVL in the sense that on traversing from node d to a, the voltage rose from 0 to 5.000V, then on traversing each 1kΩ resistor the voltage dropped by 1.667 V returning to 0V at d. Algebraically on probing clockwise around the closed loop from d to d we have + 5.000V - $1.\overline{666}$V - $1.\overline{666}$V - $1.\overline{666}$V = 0 as required by KVL.

Since the simulation satisfies KVL and resistor series summation *by design*, this agreement should be no surprise. More significant is whether a physical circuit obeys these principles. To test this the circuits were built on a breadboard powered by the 5V terminal of a DC power pack. There were insufficient 1kΩ resistors and no 2kΩ resistors in the stock so different resistor values were used in place of the resistors of Figure 1A and 1B. In any case the resistor values are only nominal resistances with as much as a 10% deviation from those values. Similarly, the 5V terminal of the power brick yielded a slightly higher voltage. The nominal voltage and resistors used, as well as the measured actual values of these used in each physical circuit are given in Table 1.

The voltage measured at the corresponding node b in the physical circuit was 3.655V while that measured at point f was 3.598V. This discrepancy is due to the differences in the resistor values, which can be confirmed as follows: using the series resistance summation of the resistors in each circuit to get the equivalent resistances allows the use of Ohm's law[1] to extract the current in each series circuit. That current can then be used to determine the voltage drops across R1 and R4 respectively. Finally, using KVL the voltages at nodes b and f can be determined.

Component	Nominal value	Measured value
V1	5V	5.166V
R1	1.2kΩ	1.116kΩ
R2	1.2kΩ	1.224kΩ
R3	1.5kΩ	1.475kΩ
R4	1.2kΩ	1.170kΩ
R5	2.7kΩ	2.685kΩ

Table 1. Actual voltage and resistor values used in the physical circuits corresponding to Figures 1A and 1B.

For physical circuit 1A this procedure gives:

$$V = I \cdot R \rightarrow I_A = \frac{V}{R_A} = \frac{5.166V}{1.116k\Omega \quad .224k\Omega \quad .475k\Omega} = \frac{5.166V}{3.815k\Omega} = 1.354mA$$

Then,

$$V_b = V_{in} - I_A \cdot R1 = 5.166V - (1.354mA)(1.116k\Omega) = 3.655V$$

While for physical circuit 1B this procedure gives:

$$V = I \cdot R \rightarrow I_B = \frac{V}{R_B} = \frac{5.166V}{1.170k\Omega + 2.685k\Omega} = \frac{5.166V}{3.815k\Omega} = 1.340mA$$

And,

$$V_f = V_{in} - I_B \cdot R4 = 5.166V - (1.340mA)(1.170k\Omega) = 3.598V$$

The perfect agreement with the values measured at the corresponding nodes in the physical circuit is a little surprising insofar as our calculations did not include any contact or wire connector resistances, but these were apparently negligible.

For physical circuit 1A the voltage drops measured across the resistors R1, R2 and R3 were: 1.511V, 1.658V and 1.997V, respectively, which summed to precisely the power supply voltage of 5.166V, consistent with KVL.

2. Parallel Resistors and KCL

As a test of the combination of parallel resistances and KCL the circuit shown in Figure 2 was modeled, built and measured. In this case the measured resistances used in the physical circuit (shown in Fig. 2) were incorporated in the LTSpice model.

The parallel combination of R2 and R3 is given by:[2]

$$R_{23} = \frac{R2 \cdot R3}{R2 + R3} = \frac{4.655k\Omega \cdot 4.720k\Omega}{4.655k\Omega + 4.720k\Omega}$$

$$R_{23} = 2.344k\Omega$$

This equivalent resistance in series with R1 yields a total resistance

$$R = R1 + R_{23} = 4.621k\Omega + 2.344k\Omega = 6.965k\Omega$$

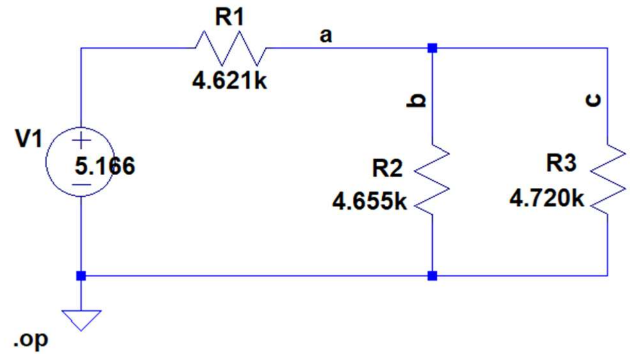

Figure 2. The circuit used to confirm the combination of parallel resistors and KCL.

This should result in a current through R1 of

$$I_a = \frac{V1}{R} = \frac{5.166V}{6.965k\Omega} = 0.742mA$$

The current through R1 was measured by breaking the circuit at node a, just after R1 but ahead of the junction, and allowing the current to pass through the multimeter used as an ammeter. The current measured was 0.74mA. The currents were similarly measured in the legs of R2 and R3 by inserting the ammeter at nodes b and c, respectively, yielding currents of 0.37 mA through each leg. KCL requires that the currents through any node sum to zero.[2] Considering the junction of legs a, b and c a node we find that,

$$I_a + I_b + I_c = -0.74mA + 0.37mA + 0.37mA = 0$$

As required by KCL.

The nearly equal, parallel resistors shown in Fig. 2 were replaced by a nominally 10kΩ and a nominally 1kΩ resistor as shown in Figure 3. The voltage at node a, measured using the oscilloscope and a 1x probe was found to be 895mV. This placed ΔV = 5.166V-0.895V = 4.271V across R1 implying that the current into the junction was $I = \frac{\Delta V}{R1} = \frac{4.271V}{4.621k\Omega} = 0.925mA$. The 0.895V across the 9.951kΩ R2 and across the 1.072kΩ R3 imply currents of

$$I_{R2} = \frac{V_a}{R2} = \frac{0.895V}{9.951k\Omega} = 0.0899mA$$

$$I_{R3} = \frac{V_a}{R3} = \frac{0.895V}{1.072k\Omega} = 0.8349mA$$

through the respective resistors (legs). In agreement with KCL these sum to equal the current through R1.

It can be noted from the results of these parallel resistor circuits that when the resistors are equal, the current flows equally through each leg (neither resistor dominating) while when one resistor is much larger than the other the major fraction of the current flows through the smaller of the two.

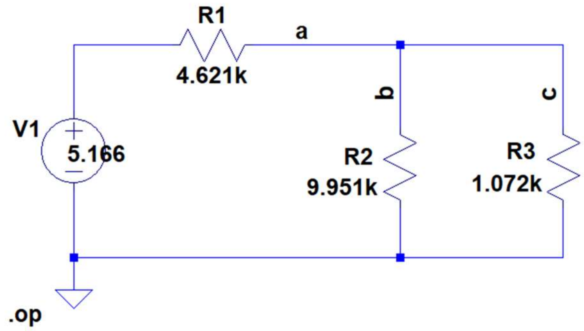

Figure 3. The circuit of Fig. 2 modified with a nominally 10k and 1k resistor in parallel.

For purposes of rapid estimation it may be useful to consider these extremes more generally. For parallel resistors A and B:

$$R_{eq} = \frac{R_A \cdot R_B}{R_A + R_B}$$

If $R_A = x \cdot R_B$, then,

$$R_{eq} = \frac{R_A \cdot R_B}{R_A + R_B} = \frac{x \cdot R_B \cdot R_B}{x \cdot R_B + R_B} = \frac{x \cdot R_B^2}{(x+1)R_B} = \frac{x}{x+1} R_B$$

If x = 1 (i.e. the resistors are equal),

$$R_{eq} = \frac{x}{x+1} R_B = \frac{1}{2} R_B$$

The equivalent resistance evaluates to *half* of either.

While if say x =10 (R_A much larger)

$$R_{eq} = \frac{x}{x+1} R_B = \frac{10}{11} R_B$$

The equivalent resistance is less than, but approaches the *smaller* of the two resistances.

3. Thevinin's Theorem

Thevenin's equivalent circuit theorem says that any complex, two terminal network of power supplies and resistances can be reduced to a single (ideal) power supply operating at a voltage V_{th} (the Thevinin voltage) through a single series resistance R_{th} (the Thevinin resistance).[1] As a test of this principle the circuit shown in Figure 4 was modeled, built and tested.

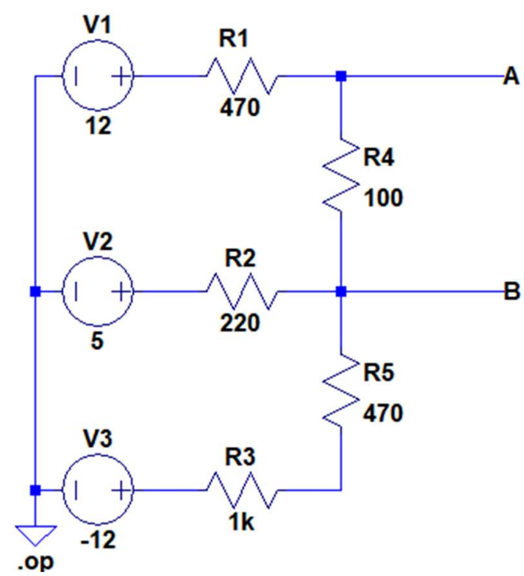

Figure 4. Circuit used in a test of Thevenin's theorem.

The nominal voltages and resistances were used in the simulation, accepting that small differences between the model and the measurements were due to the tolerances in the actual components.

V_{th} is the open circuit voltage measured between nodes A and B. The simulation returned for the voltage at node A 6.313V and for the voltage at node B 5.103V, their difference being V_{th} = 1.210V. Measurement of the potential difference between nodes A and B in the physical circuit using the multimeter gave a very similar voltage of 1.221V. To determine R_{th} by simulation a resistor was placed between nodes A and B and its resistance was made a very small 0.001Ω, effectively shorting across the two nodes. Running an operating point simulation yielded the short circuit current, I_{sc} = 13.93 mA through this negligible resistance, from which R_{th} was determined using,

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{1.210V}{0.01393A} = 86.86\Omega$$

To do this in the physical circuit, recalling that the ammeter is a low impedance instrument, it was placed across the two nodes and the short circuit through it measured. This yielded a current of 13.71mA giving for the physical circuit an R_{th} = 89.06Ω, close to the simulation result.

Within Thevenin's theorem, placing a 220Ω load resistor across nodes A and B is equivalent to the circuit shown in Figure 5. For the simulation this gave a current through the 220Ω load of 3.94 mA, which was measured in the physical circuit to be 3.95mA. This close agreement provides good support for the validity of Thevinin's theorem.

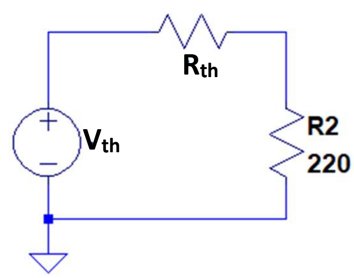

Figure 5. Thevenin's equivalent circuit.

4. An optically coupled circuit

Figure 6 shows the circuit built. This consisted of a green light emitting diode (LED) powered by the HP variable DC power supply, current limited by a series 470Ω resistor (protecting the LED). The LED was oriented to shine its light onto the face of VT90N1 photoresistor, with the two (largely) isolated from ambient light by an opaque enclosure. The photoresistor was powered by a 12V supply and was wired in series with a 10kΩ resistor to ground. In this configuration the photoresistor acted as the variable resistance upper leg of a voltage divider with the 10kΩ resistor the lower leg. The resistance of the photoresistor depending on the intensity of light that impinged on it. Table 2 shows the data recorded and result of calculations, wherein:

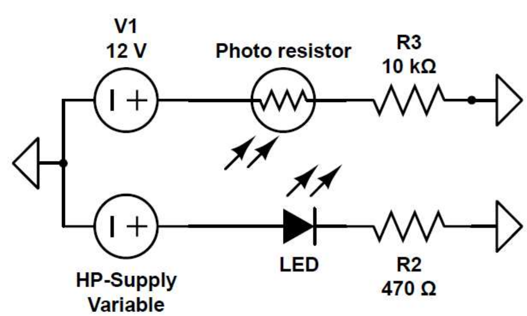

Figure 6. A light emitting diode exciting a photoresistor.

V_{HP} : HP voltage powering the LED

V_{R2} : measured voltage across R2 (R2 actual value 463Ω)

I_{R2} : current through R2 (and the series LED) calculated by dividing V_{R2} by the 463Ω R2

V_{R3} : voltage measured across R3 (R3 actual value 9965Ω)

$R_{photores}$: Resistance of the photoresistor calculated from V_{R3}, as discussed below.

The HP power supply also reported the current it was sourcing to the LED, but its resolution was only 1 mA so it could not be very accurate. Measuring V_{R2} let that current be calculated with much greater precision (column 3, Table 2).

V_{HP} (V)	V_{R2} (V)	I_{R2} (A)	V_{R3} (V)	$R_{photores}$ (Ω)
0	0	0	0.740	151,600
1	0	0	0.740	151,600
2	0.184	0.000397	0.846	131,400
3	1.12	0.00242	0.982	111,800
4	2.08	0.00449	1.175	91,810
5	3.04	0.00657	1.465	71,660
6	4.03	0.00870	1.930	51,990
7	4.99	0.0108	2.852	31,960

Table 2. Data and calculations associated with the circuit of Figure 6 (see text for definitions).

Figure 7 plots the current through the LED as a function of the HP voltage.

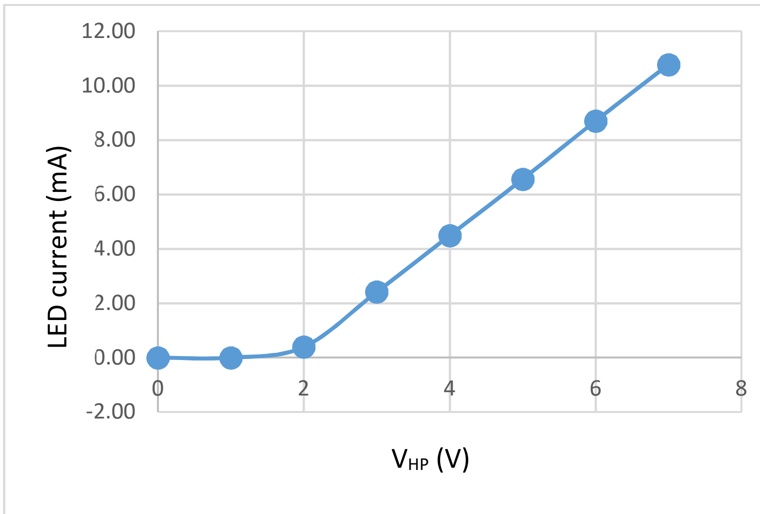

Figure 7. Current through the LED plotted versus V_{HP}.

For any given light intensity impinging on it the photoresistor has some specific resistance, $R_{photores}$. Because it is acting as the upper leg of a voltage divider with R3 the lower leg, the voltage across R3 should be:

$$V_{R3} = 12V \cdot \frac{R3}{R_{photoresis} + R3}$$

Using the measured voltages V_{R3} this could be inverted, solving for $R_{photores}$, giving:

$$R_{photores} = R3 \cdot \frac{12V - V_{R3}}{V_{R3}}$$

Which yielded the last column in Table 2.

With better ambient light rejection the utility of such a set-up is the electrical isolation of the photoresistor side of the circuit from the light emitting side.[3] In practice this could be used to protect expensive, sensitive, instrumentation from electrical spikes that may occur on the LED side of the circuit.

References:

1. LTSpice Electronic Circuit Simulation Software is available for free from Analog Devices at: https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html

2. Basic Electronics for scientists and engineers, Dennis L. Eggleston, Cambridge University Press, New York, 2011.

3. Wikipedia: https://en.wikipedia.org/wiki/Opto-isolator